1 Reflection & Shadows

1. Rotating mirror. For a cuboid with a square base $ABCD$, two of the faces are, respectively, a screen AB and a plane mirror AD. A small plane mirror M rotates uniformly about the edge C with a period $T = 12\,\text{min}$. A laser emits from a small hole at the edge A (Figure 2). For a given full rotation of the mirror, find the interval of time t during which the laser's dot Z will move along AB. *(Russia, 2006-R-9)*

2. A room with mirrors. Two floor-to-ceiling mirrors of width $c = 1\,\text{m}$ each are hung in the corner of a room. The dimensions of the room are $a \times b \times H = 9\,\text{m} \times 3.5\,\text{m} \times 4\,\text{m}$. A point source is placed at a distance c from both mirrors, and set up so that it illuminates only the mirrors (Figure 1). Do any parts of the walls remain unilluminated? If yes, what is their total area? *(Russia, 2012-R-9)*

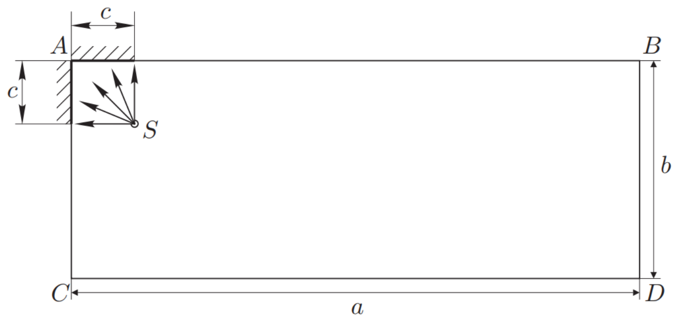

Figure 1

3. Lost mirror. They say that someone found a diagram in Snell's archive. It depicts two semi-infinite plane mirrors M_1 and M_2 at an angle φ to each other, a point source S, and the region AOB from which one can observe both images of the source S simultaneously. With time, the ink has faded, and it has become impossible to discern the positions of the mirror M_2 and the source S (Figure 3). Using a straightedge and compass, recover M_2 and the locus of S. Calculate φ for $\angle AOB = \alpha = 30°$. *(Russia, 2012-F-9)*

4. Solar position. Recover the Sun's position and the top edge of the fence on Figure 17. Justify your constructions. *(Russia, 2015-R-9)*

5. IPhO Camp. Alice and Bob went to the IPhO Training Camp on an island on the equator. Two 100-floor buildings have been constructed, one to the east of the other. The buildings lie parallel to each other and perpendicularly to the equator (Figure 4). Alice's room is in the western building, while Bob's room is on the 10-th floor in the eastern building, and their windows happen to look out onto each other. On the day of the vernal equinox, March 21, the Sun shines into Bob's window for $T_1 = 2\,\text{hrs}$, and into Alice's window for $T_2 = 4\,\text{hrs}$.

1. On what floor is Alice's room?
2. At what time does Alice first see a glow in Bob's window?
3. For how long will Alice see a glow in Bob's window?

The windows "glow" when they reflect sunlight towards the observer. Do not consider multiple reflections. The Sun reaches the zenith at 12 o'clock. *(Russia, 2015-F-9)*

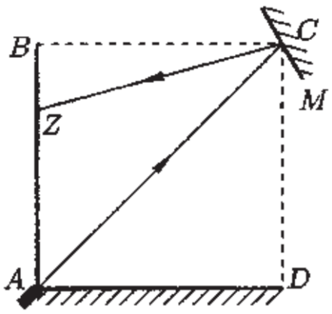

Figure 2

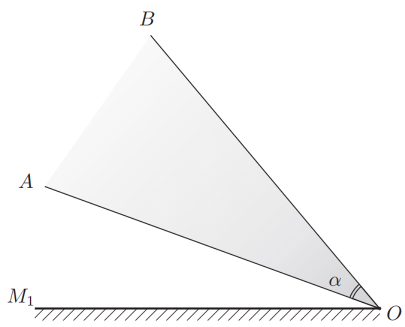

Figure 3

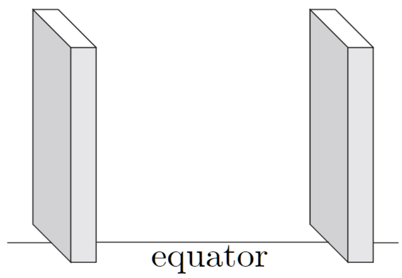

Figure 4

2 Snell's Law

6. Laser in vessel. A small laser is placed at a point A inside a thin cylindrical glass vessel which is suspended in air. The vessel is filled with water, the refractive index of which is $n = 4/3$. Half the internal surface of the cylinder, corresponding to the arc ACB, is painted black and absorbs all light. The laser initially points towards B. The laser begins to rotate counterclockwise around the axis passing through A with a constant angular velocity ω (Figure 5).

1. Find the time τ until the laser beam ceases to exit the vessel.
2. What is the velocity of the laser's dot on the painted surface of the cylinder at time 1.5τ?

(Russia, 2017-F-9)

7. Tetragon. The base of a right glass prism is the symmetric quadrilateral OAO_1D shown on Figure 6. $\angle AOD$ is a right angle. A ray L_1 enters at OA normally, and following reflections at DO_1 and AO_1, it leaves at OD normally as well. Now, a ray L_2 enters at OA at an angle α. At what angle β relative to the normal of OD does this ray leave following reflections at DO_1 and AO_1? *(Russia, 2019-R-9)*

8. Living in a bubble. There is a planet in outer space that consists entirely of water. Its marine inhabitants can see beyond the ocean in all directions only when they are less than $x = 3000\,\text{km}$ away from the centre of the planet. This civilisation has launched an artificial satellite on the lowest possible orbit around the planet. Find the velocity of the satellite. The refractive index of water is $n = 4/3$, the density of water is $\rho = 1000\,\text{kg/m}^3$, the gravitational constant is $G = 6.67 \times 10^{-11}\,\text{N}\,\text{m}^2/\text{kg}^2$. The planet does not rotate around its axis and there are no waves on its surface. Assume that the water is incompressible. *(Russia, 2018-F-11)*

9. Lunar eclipse. As you know, the Sun is not a point source of light – viewed from Earth, it has an angular diameter of $2\delta = 0.52°$. This implies that the Earth's umbra (Figure 7) has a finite extent.

1. Assume that there is no atmospheric refraction. Find the extent of the umbra L_1. Find the duration of a total lunar eclipse in that case. Neglect the motion of the Earth around the Sun.
2. In reality, atmospheric refraction has a considerable effect on the extent of the Earth's umbra. Consider a simple model in which the atmosphere has an effective height $h = 8\,\text{km}$ and an average refractive index $n = 1.00028$. Assuming that the limits of the umbra are formed by the rays tangential to the Earth's surface, find the extent of the umbra L_2. What fraction of the lunar disc is within the umbra?

The Earth's radius is $R = 6400\,\text{km}$, the gravity of Earth is $g = 9.8\,\text{m/s}^2$, the Moon's orbital period is $T_0 = 27.3\,\text{d}$. The Moon's angular diameter is equal to that of the Sun, 2δ. *(Russia, 2017-F-11)*

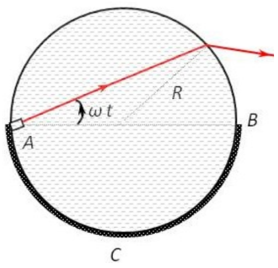

Figure 5

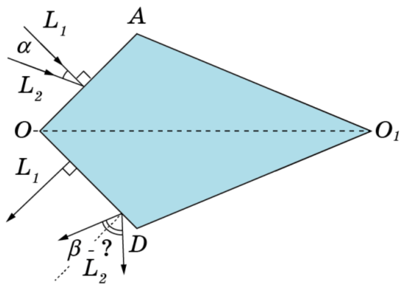

Figure 6

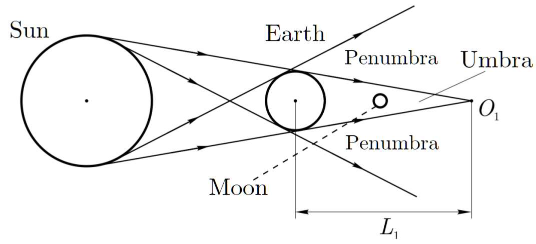

Figure 7

10. Mermaid. A prince, $H = 1.8\,\mathrm{m}$ tall, stood on the ocean's shore, looking at the track of moonlight in the water (Figure 8). The track began $D_P = 5\,\mathrm{m}$ away from him (horizontally), and had length $L_P = 50\,\mathrm{m}$. In the water, a mermaid lay along the shore at depth H, also entranced by the moonlight.

1. At what horizontal distance D_M away from the mermaid does the track of moonlight, as observed by her, begin?
2. What is the length of that track L_M?

Assume that a light breeze is creating a uniform swell on the water's surface. The refractive index of seawater is $n = 1.35$. Neglect the angular size of the Moon. *(Russia, 2019-F-11)*

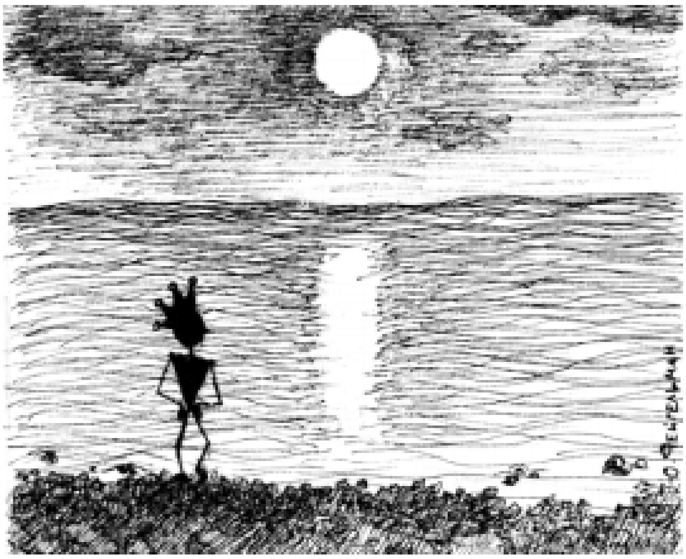

Figure 8

3 Lenses

3.1 Constructions

11. Lost ray. They say that someone found a diagram in Snell's archive. It depicts a thin convex lens, its foci, and a ray that passes through the lens. With time, the ink has faded, and only two points on the ray, A and B, remain (Figure 9). Recover the ray. *(Russia, 2002-R-9)*

12. Refracted ray. They say that someone found a diagram in Snell's archive. With time, the ink has faded, and only three points remain – the right focus F of a thin lens, the point A, at which an incoming ray AA' refracts, and a point B on the left focal plane of the lens (Figure 10). Recover the plane of the lens, its optical axis, and the ray after its refraction. *(Russia, 2009-F-10)*

13. Three points. They say that someone found a diagram in Snell's archive. With time, the ink has faded, and only three points remain – the centre of a lens O, a point on its front focal plane A, and a point on its back focal plane B. The points A and B belong to a ray that passes through the lens (Figure 11). Recover the ray, the plane of the lens, and the foci. *(Russia, 2002-R-11)*
Hint: A purely geometrical solution is possible, and is the most pleasing, but the thin lens formula can often help in such problems.

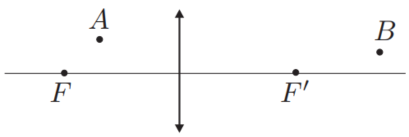

Figure 9

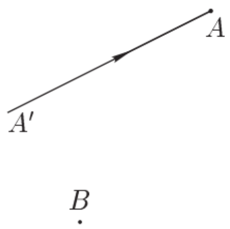

Figure 10

Figure 11

14. Two arrows. On Figure 12 there is an object AB and its image $A'B'$ in a thin lens. Recover:

1. The centre of the lens;
2. The plane of the lens;
3. The foci of the lens.

Is the lens concave or convex? *(IZhO, 2011)*

15. Lens recovery. They say that someone found a diagram in Snell's archive. It depicts a thin lens, an object and its image. The object is a rod of length l with two point sources at its ends. The rod and the optical axis of the lens are in the plane of the diagram, and the rod does not cross the plane of the lens. With time, the ink has faded; only the sources and their images remain, and one cannot discern which is which. These four points are located on the vertices and the centre of an equilateral triangle (Figure 13).

1. Determine whether the point at the centre of the triangle belongs to the object or to the image.
2. Recover the optical setup (the object, the image, the lens, the optical axis of the lens, the foci), accurate to a reflection and to a 120° rotation.
3. Find the focal length of the lens.

(Russia, 2013-F-11)

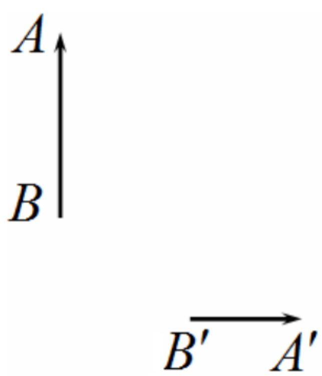

Figure 12

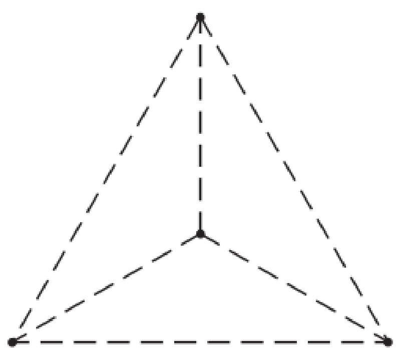

Figure 13

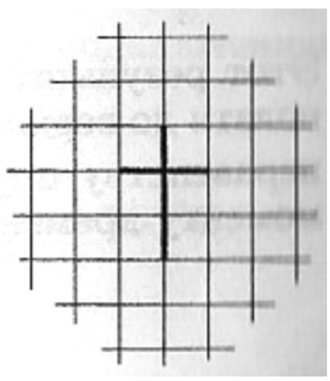

Figure 14

3.2 The Thin Lens Formula

16. Lens and cross. They say that someone found a diagram in Snell's archive. It depicts a thin lens, an object and its image. With time, the ink has faded, and only the object remains on a square grid (Figure 14). The object and the image are of the same size and shape, and the optical axis is parallel to some of the gridlines. Recover the optical setup (the object, the lens, the foci). *(Russia, 2002-F-11)*

17. A pair of lenses. Two thin lenses L_1 and L_2 with focal lengths F_1 and F_2 are at a distance L from each other. A thin lens L_3 is placed between L_1 and L_2 such that any ray entering the optical setup at a small angle to the optical axis will leave parallel to its initial direction. Find the focal length F_3 and the distances l_1 and l_2 from L_3 to L_1 and L_2, respectively. The optical axes of three lenses coincide. *(Russia, 2007-R-11)*
Note: The focal lengths given may be positive or negative, depending on whether the lenses are convex or concave.

18. Millicar. A very small car, the size of an ant, moves on a horizontal surface along the optical axis of a convex lens whose focal length is f. The coefficient of friction between the wheels of the car and the surface is μ. A point source S is attached to the car. The velocity of the car changes in such a way that the velocity of the image S_1 remains constant and equal to v_0. At what distances to the lens is such motion achievable? *(Russia, 2015-F-11)*

3.3 Lens Fundamentals

19. Silvered lens. The spherical surface of a plano-convex lens of focal length F_1 was silvered. If a beam is directed along the optical axis and towards the spherical surface, the reflected rays will seem to emanate from a point F'' located F_2 away from the lens (Figure 15). A beam is now directed along the optical axis and towards the planar surface. Find the respective focal length F_0, i.e. the distance between the lens and the point at which this beam is focused. The focal length of the lens is much larger than its diameter, and the silvered surface reflects all incoming radiation. *(Russia, 2004-R-10)*

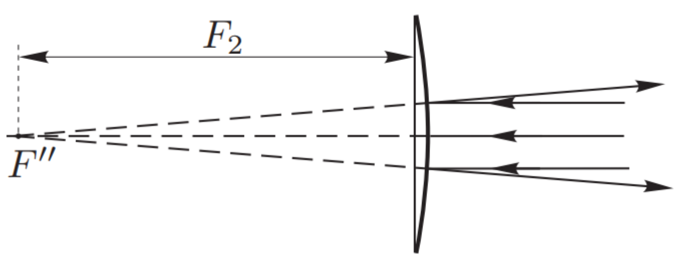

Figure 15

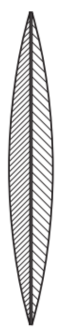

Figure 16

20. Composite lens. An optical setup consists of two equiconvex[1] lenses L_1 and L_2. A parallel beam that enters the setup (passing through L_1 first) will leave as a parallel beam that is γ times as wide. If L_1 and L_2 are submerged in glycerine, they will remain converging, but their focal lengths will increase by a factor of α and β, respectively. The lenses are split into two equal halves each, and a composite lens is assembled (Figure 16). If the composite lens is submerged in glycerine, by what factor will its focal length increase? *(Russia, 2004-R-11)*

[1]Having two convex surfaces whose radii of curvature are equal.

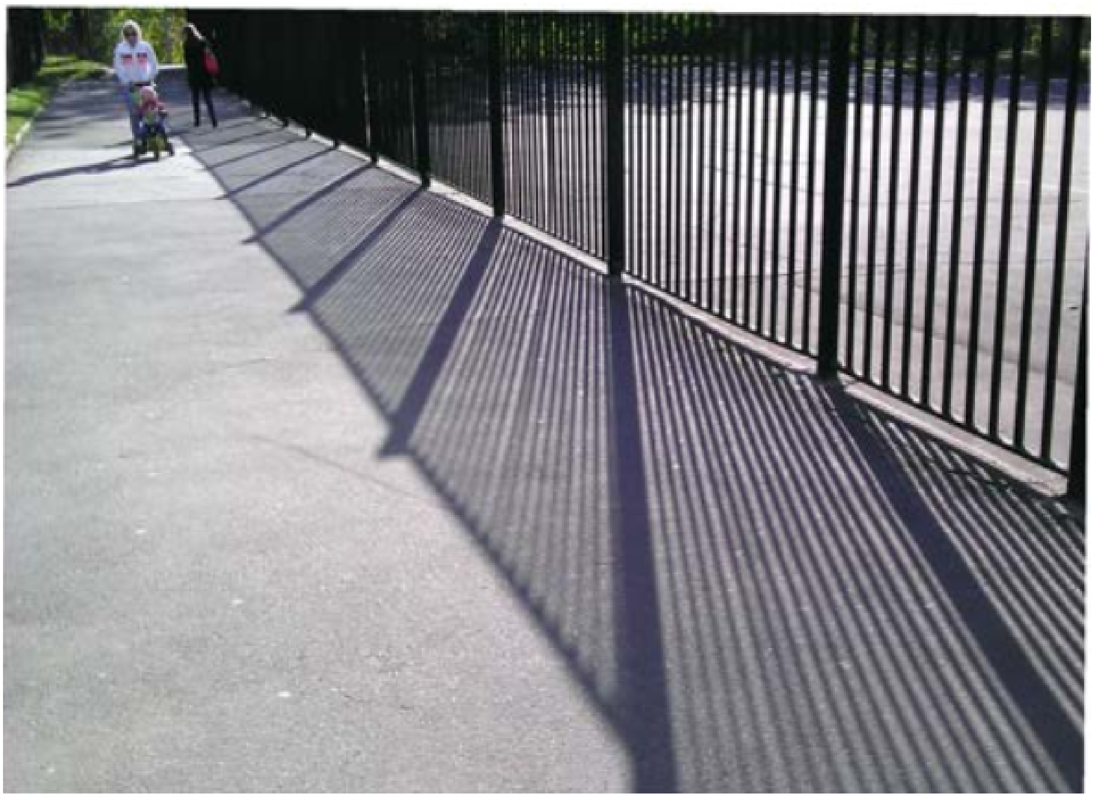

Figure 17