2026 Bulgarian IPhO Team Selection Test

Short Exam 1

Problem. "Stationary" orbit. A small body can slide without friction along the inner surface of a fixed sphere of radius R. The position of the body is described by the angle θ between its radius vector and the vertical line passing through the centre, with $\theta = 0$ corresponding to the lowest point of the sphere. The acceleration due to gravity is g.

(a) The body is placed at a point with a deviation angle θ_0 from the vertical, as shown on Figure 1 ($\theta_0 < \pi/2$). What initial horizontal velocity v_0 must be imparted to the body so that it continues to move along a horizontal circle, i.e. along the parallel through the initial point? **(1.5 pt)**

(b) Suppose that at the initial moment, in addition to the horizontal velocity $\vec{v}_0$ determined in part (a), the body is also given some small extra velocity $\vec{v}_1$ ($v_1 \ll v_0$) in the direction of the meridian on which it is placed. For what initial angle θ_0 will the body return to the starting point after exactly one lap, with a speed identical in magnitude to the initial speed? **(3.5 pt)**

Time: 45 minutes

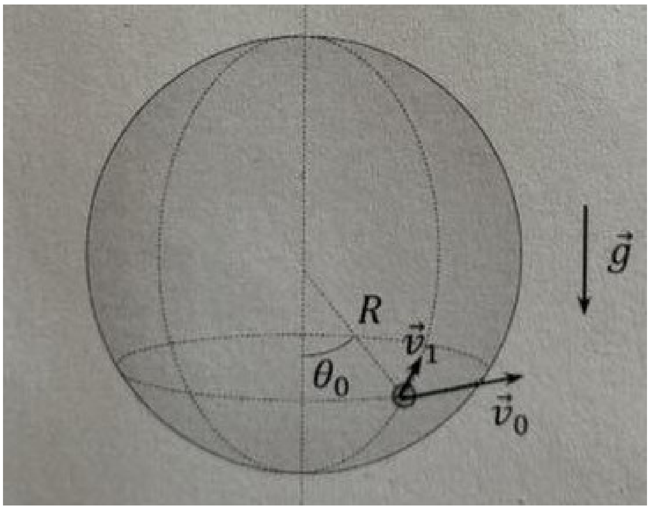

Figure 1

Short Exam 2

Problem. Transparent sphere. A transparent sphere of refractive index n has radius R. Inside it, at a distance r from its centre, there is a point source of light.

(a) What fraction k of the light beam emitted by the source will undergo total internal reflection? What are the constraints on r for total internal reflection to be observed?

(b) A spherical screen of radius L is placed around the sphere. The sphere and the screen are concentric. What must L be so that a bright glowing point can be observed on the screen? What are the constraints on n and r for this to occur?

Time: 50 minutes

Short Exam 3

Problem. Diamagnetism. Consider an electron moving uniformly in a circle (for example, on a circular orbit around the stationary nucleus in a hydrogen atom). This system has a magnetic dipole moment $\mu = IS$, where I is the current associated with the motion of the electron along the circle, and S is the area of the circle bounded by this orbit.

(a) Find the relationship between the magnetic dipole moment and the angular momentum of the electron.

The system is placed in a magnetic field B perpendicular to the plane of the circle.

(b) Find the change in the magnetic moment when the magnetic field changes from 0 to B. Assume that the radius of the orbit does not change.

In the formula you've obtained, the radius of the circle appears as ρ^2, whereas the quantum mechanical formula involves the expectation value $\langle\rho^2\rangle$ instead. The ground state of the hydrogen atom is spherically symmetric, and therefore the following relation holds:

$$\langle\rho^2\rangle = \frac{2}{3}\langle r^2\rangle,$$

where r is the distance from the electron to the nucleus in three-dimensional space. The magnetic moment of one mole of matter can be obtained by summing over all electrons in a given atom and then over all atoms. The ratio of the magnetic moment ΔM to the magnetic induction B is called the magnetic susceptibility χ, with

$$\chi = \mu_0 \frac{\Delta M}{B}.$$

(c) Using the experimental value of the molar magnetic susceptibility of helium in the gaseous state ($\chi = -2.36 \times 10^{-11}\,\mathrm{m^3/mol}$), estimate the root-mean-square distance between the electron and the nucleus in the ground state of the helium atom.

Time: 60 minutes

Theoretical Exam

Problem 1. Channel. A channel with a rectangular cross-section is confined by a concrete slab of height $a = 2.0\,\mathrm{m}$ and width $b = 0.2\,\mathrm{m}$, as shown on Figure 2. The coefficient of friction between the slab and the bottom of the channel is $k = 1$. The density of concrete is $\rho_1 = 2.3 \times 10^3\,\mathrm{kg/m^3}$ and the density of water is $\rho_0 = 1.0 \times 10^3\,\mathrm{kg/m^3}$. Find the maximum water level H that the slab can hold.

Problem 2. Oscillating rods. Two identical uniform rods, each of mass m and length l, are connected by a hinge so that they can rotate relative to each other without any friction. The free ends of the rods are connected by a spring with constant k, such that at equilibrium the rods form a right angle (Figure 3). The system is in weightlessness.

The rods are displaced in opposite directions from their equilibrium position by a small angle, and then are released from rest. Find an expression for the frequency ν of the resulting oscillations of the system.

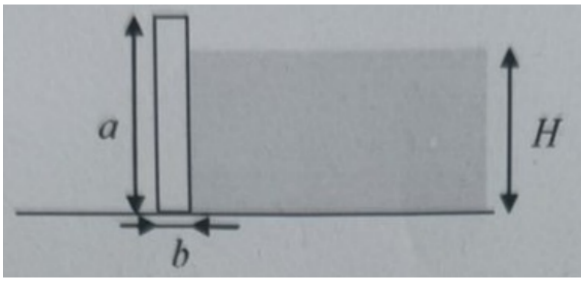

Figure 2

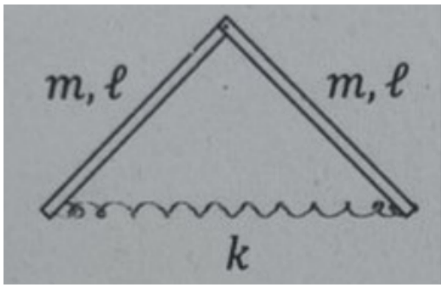

Figure 3

Problem 3. Collision. A gamma-ray photon interacts with a stationary proton, producing a π^0 meson:

$$\gamma + p \to p + \pi^0$$

What is the minimum energy of the gamma-ray photon for this process to be possible? The rest masses of the proton and the pion are $m_p = 938\,\mathrm{MeV/c^2}$ and $m_\pi = 135\,\mathrm{MeV/c^2}$.

Problem 4. Faraday homopolar generator. An ideal conducting disk of radius r_0 is placed in a constant uniform magnetic field with induction B, perpendicular to the disk. A resistor with resistance R is connected between the centre and the rim of the disk. A body of mass M is attached to a thread wound around the disk, as shown on Figure 4. The gravitational acceleration is g. The body begins to fall, and after some time the disk reaches its maximum angular velocity. Find this angular velocity ω and the current I flowing through the resistor.

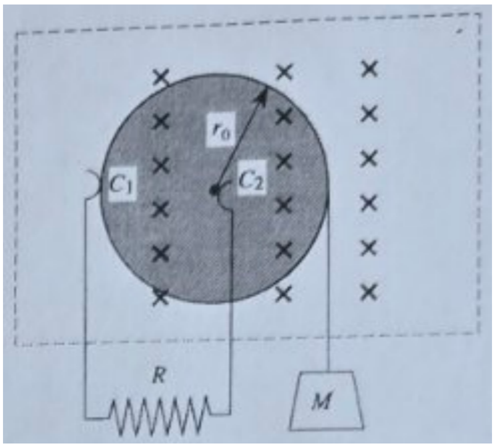

Figure 4

Problem 5. AC bridge. An AC bridge (Figure 5) consists of a source of alternating voltage $V(t) = U_0 \cos(\omega t)$, two identical resistors, each with resistance R, a capacitance C, and an inductance L. An oscilloscope connected at the middle terminals of the bridge then measures a voltage $U(t) = U_1 \cos(\omega t + \varphi)$.

(a) For what relation between the parameters of the bridge do we get $U_1 = 0$?

(b) For arbitrary parameters of the bridge, there is some frequency at which the voltages $V(t)$ and $U(t)$ are in phase (i.e. $\varphi = 0$). Find this frequency ω_0. At this ω_0, what is the ratio U_1/U_0?

Problem 6. Thermal radiation. Consider a body in thermodynamic equilibrium with an evacuated cavity inside it. The energy density of the radiation inside the cavity is u, and the intensity of the thermal radiation incident on the cavity wall (energy per unit area per unit time) is Φ. Express Φ in terms of u.

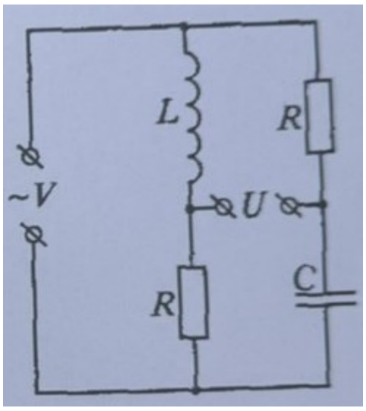

Figure 5

Problem 7. Pistons. In a long horizontal cylindrical tube there are two pistons of masses $M_1 = 2\,\mathrm{kg}$ and $M_2 = 1\,\mathrm{kg}$, which can move with virtually no friction. Between the pistons there is one mole of ideal gas (helium). We apply opposing forces $F_1 = 100\,\mathrm{N}$ and $F_2 = 50\,\mathrm{N}$ to the pistons, as shown on Figure 6. What is the equilibrium distance l between the pistons? The temperature of the gas T is constant and equal to $10\,\mathrm{K}$. The tube is in vacuum.

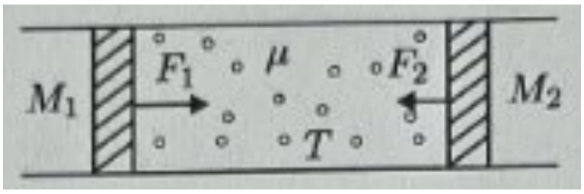

Figure 6

Problem 8. Photon in a cylinder. Consider a cylindrical chamber with a piston where both the cylinder base and the piston have a mirror coating. The chamber contains a single photon of frequency ω_0 whose momentum is directed perpendicularly to the piston. The piston begins to move slowly at constant speed until the volume of the container is reduced by a factor of k. Find the final frequency of the photon. Assume that the wavelength of the photon is much smaller than the dimensions of the chamber, and that the momentum of the photon is much smaller than that of the piston.

Problem 9. CO_2 laser. The CO_2 molecule has a large number of closely spaced discrete energy levels, transitions between which can lead to the generation of laser radiation with a wavelength $\lambda \approx 10\,\mu\mathrm{m}$ and a spacing between the spectral lines $\Delta\lambda \approx 20\,\mathrm{nm}$. To achieve continuous tuning of the laser frequency, we increase the pressure in the cavity. The resulting broadening of the levels is governed by the intermolecular collisions, whose rate depends on the mean free time. The broadening causes the lines to merge into a single band.

Estimate the pressure P at temperature $T = 400\,\mathrm{K}$ for which this merging becomes possible. The collision cross-section between two molecules is $\sigma \approx 10^{-19}\,\mathrm{m}^2$.

Problem 10. Bose-Einstein condensate. An ideal monatomic gas of bosons ($^4_2\mathrm{He}$) is cooled down at constant volume V and constant particle number N. As its temperature decreases, we reach a temperature T_0 below which the properties of the gas arise from the quantum properties of the bosons – i.e. their wavelike nature and their indistinguishability.

(a) Find T_0. The wavelike properties become significant when the de Broglie wavelength at the average thermal energy is approximately equal to the mean distance between the particles. Provide a numerical estimate for the number density $n = N/V$ if $T_0 = 4\,\mathrm{K}$.

At temperatures $T < T_0$ the particles of the gas can be separated into two groups, each encompassing a nonnegligible number of particles. The first group consists of N_0 particles at the lowest energy level ($\varepsilon = 0$), which do not take part in the thermal motion. The second group consists of N^* particles distributed across various energy levels (with $\varepsilon > 0$). These do take part in the thermal motion, and their number is given by

$$N^* = N \left(\frac{T}{T_0} \right)^{3/2} .$$

This is called a degenerate Bose gas.

(b) Find the heat capacity of the gas C_V when $T < T_0$.

(c) Find the pressure of the gas P when $T < T_0$. What is interesting about this result?

Apart from the thermodynamic variables T, V, and N, your results must include the Planck constant h, the Boltzmann constant k_B, and the mass of the helium atom, $m = 6.7 \times 10^{-27}\,\mathrm{kg}$.

Experimental Exam

Problem 1. Measuring g with a ruler and a stopwatch.

Equipment:

1. Ruler with millimetre graduations and a plastic end-cap with a bearing fixed to it, for mounting the ruler on a rotation axis (Figure 7). The ruler may also be used for drawing.

   Note: The ruler and the end-cap form a single body that must not be disassembled. From here on, this composite body is just referred to as the ruler.

2. L-shaped steel bracket with an axis (bolt) for mounting the ruler.

3. Clamp for attaching the bracket to the table.

4. Stopwatch. The stopwatch is started by pressing the red button (on the right) and stopped by pressing it again. The display is reset using the black button on the left. If the stopwatch is off at the start of the experiment, hold down the red button until a zero reading appears on the display.

5. Two magnets of mass $m = 11.7\,\mathrm{g}$ each, to serve as weights. The magnets can be attached simultaneously on both sides of the ruler with their opposite poles facing each other.

6. Two sheets of graph paper.

7. Blank paper.

You may assume that the surface of the table is horizontal and its edges are perpendicular to each other.

Tasks:

(a) Let us take the scale of the ruler as the x-axis. Provide theoretical justification and take measurements so as to determine the mass M of the ruler and the coordinate x_C of its centre of mass.

(b) Provide theoretical justification and take measurements to determine the moment of inertia I_C of the ruler about its centre of mass, as well as the value of the acceleration due to gravity g.

For both tasks, present your measurements and calculations using tables and graphs where appropriate.

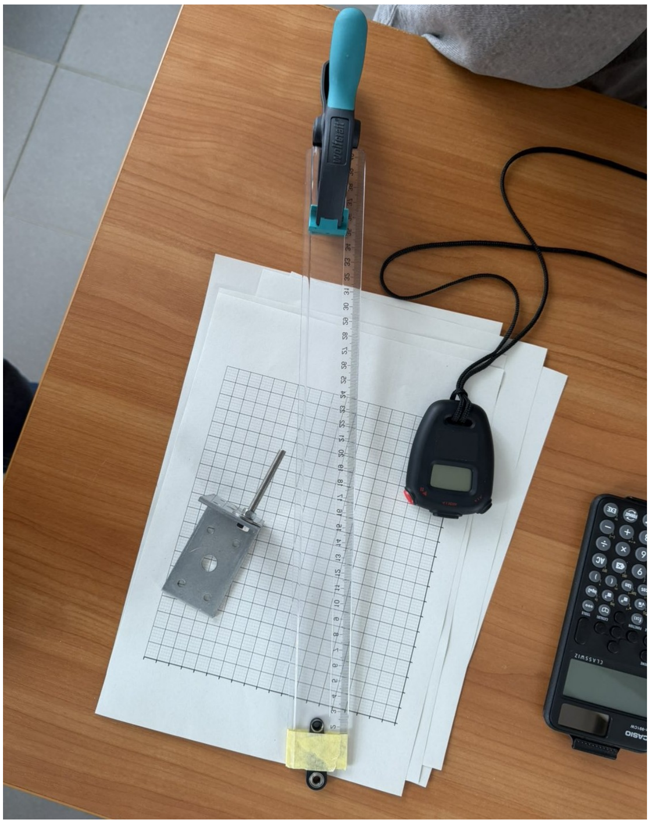

Figure 7

Note:

1. If you are unable to determine the acceleration due to gravity, you can still find I_C after taking $g = 9.81\,\mathrm{m/s^2}$ as given. In this case, however, you will lose a significant portion of the marks for the second subpart.

2. Do not attempt to fabricate data when determining g! If you do this, you will get **zero** marks for the second subpart.

Problem 2. Diode and resistor circuit.

Equipment:
Circuit consisting of two identical diodes and a resistor (Figure 8; the diodes are connected in parallel with opposite orientations, and the resistor is in series with one of the diodes), rectifier which can supply either constant voltage or constant current, two multimeters, resistor substitution box (current not to exceed 100 mA), wires, graph paper.

Task 1. Finding the resistance of the resistor R. **(7.0 pt)**
In this part of the problem you will measure the I-V curve of the circuit (without using the substitution box) for both positive and negative (i.e. with reversed polarity) voltages.

Note: Do not exceed a current of 3.0 A.

(a) Sketch the circuit that you have assembled.

(b) Write down the ranges that you use for the multimeters.

(c) Describe how R can be calculated from your measurements.

(d) How will you use the rectifier – to supply a constant voltage or to supply a constant current?

Note: The characteristics of the diodes have a strong dependence on temperature.

(e) Measure the I-V curve of the circuit as the voltage/current is raised. After you have reached the maximum voltage/current, wait until the open diode reaches its equilibrium temperature. Then, measure the I-V curve of the circuit as the voltage/current is lowered. Repeat this for voltages of the opposite polarity. Present your results in a table.

Note: A smell of hot plastic is to be expected, but in case of smoke, call the examiner immediately.

(f) Decide on the dataset that you will use for determining R. Choose between the values taken when raising the current/voltage and those taken when lowering the current/voltage.

(g) Plot a graph from which you can find R.

(h) Find R from the graph.

(i) Using the graph, find your error ΔR.

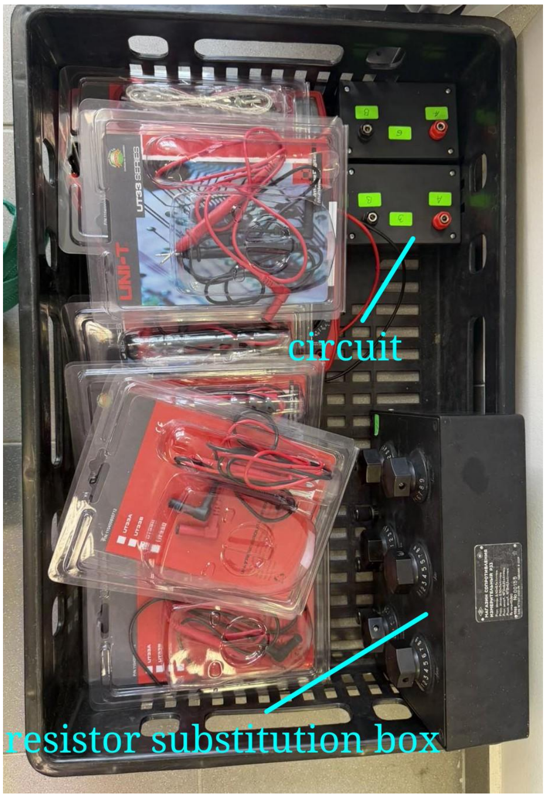

Figure 8

Task 2. Finding the reverse-bias saturation current of the diodes I_S. **(7.0 pt)**

The current I_S is the maximum current through a closed diode. The I-V curve of a diode can be modelled by the Shockley diode equation,

$$I = I_S \left(e^{\frac{eU}{nk_B T}} - 1 \right),$$

where e is the charge of the electron, k_B is the Boltzmann constant, T is the absolute temperature, and n is a number on the order of 1.

(a) Find an approximation of the formula above which can be used when measuring the forward I-V curve for voltages on the order of a few hundred mV at room temperature.

(b) Apply a voltage of such polarity that the diode with no resistor attached to it is open.

(c) Measure an appropriate part of the I-V curve for currents under 100 mA. Use the resistor substitution box if necessary. Present your results in a table.

(d) Plot your data in appropriate variables.

(e) Using the plot, find I_S and n.

Task 3. Using the data obtained so far, find the temperature of the diode T when a current $I_0 = 3\,\mathrm{A}$ flows through it. **(1.0 pt)**

Call the examiner if you suspect that a multimeter's battery has drained, that its fuse has blown, or in case of any other technical difficulties.

Constants:

Universal gas constant	R	$8.3\,\mathrm{J/(mol{\cdot}K)}$
Avogadro constant	N_A	$6.02 \times 10^{23}\,\mathrm{mol^{-1}}$
Boltzmann constant	k_B	$1.38 \times 10^{-23}\,\mathrm{J/K}$
Elementary charge	q_0	$1.60 \times 10^{-19}\,\mathrm{C}$
Electron mass	m	$9.11 \times 10^{-31}\,\mathrm{kg}$
Vacuum permeability	μ_0	$4\pi \times 10^{-7}\,\mathrm{V{\cdot}s/A{\cdot}m}$
Planck constant	h	$6.63 \times 10^{-34}\,\mathrm{J{\cdot}s}$
Reduced Planck constant	$\hbar$	$1.05 \times 10^{-34}\,\mathrm{J{\cdot}s}$